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HEATING OF A BULKY BODY BY A CIRCULAR HEAT SOURCE WITH HEAT ELIMINATION FROM THE SURFACE TAKEN INTO ACCOUNT

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Results of an analytical and numerical solution of the problem, in a form suitable for the determination of material properties, are given.

The problem of heating a bulky body by a circular heat source is a computational scheme of an enormous number of local-heating cases encountered in engineering. Included here are the electroerosive treatment of metals, electron-beam and laser treatment, welding, the action of local heat sources in a fire, and problems of many other branches of engineering. This research is performed directly in connection with the problem of determining the heat conductivity of structural constructions (panels, etc.) under nondestructive testing – the action of a circular heat source of given intensity on the surface of an item. A stationary modification of such a method is proposed in [1]. The thermal engineering basis of the nonstationary modification of the method, proposed by the same author, is examined below. Particular cases of this computational scheme were examined in [2-6].

The problem is formulated thus. The equation

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2},\quad r,\ z\geqslant 0,\quad \tau>0$$

with the boundary conditions

$$\theta=0 \text{ for } \tau=0,\ \theta\to 0 \text{ for } r,\ z\to\infty,$$
$$\mathrm{Bi}\,\theta-\frac{\partial\theta}{\partial z}=A(r,\ \tau) \text{ for } z=0$$

is solved.

In quadratures, the solution of the problem has the form

$$\theta(r,\ z,\ \tau)=\int\limits_0^\tau\int\limits_0^\infty\frac{\zeta A(\zeta,\ \tau-t)}{2t}\exp\left(-\frac{r^2+z^2+\zeta^2}{4t}\right)I_0\left(\frac{r\zeta}{2t}\right)\times$$

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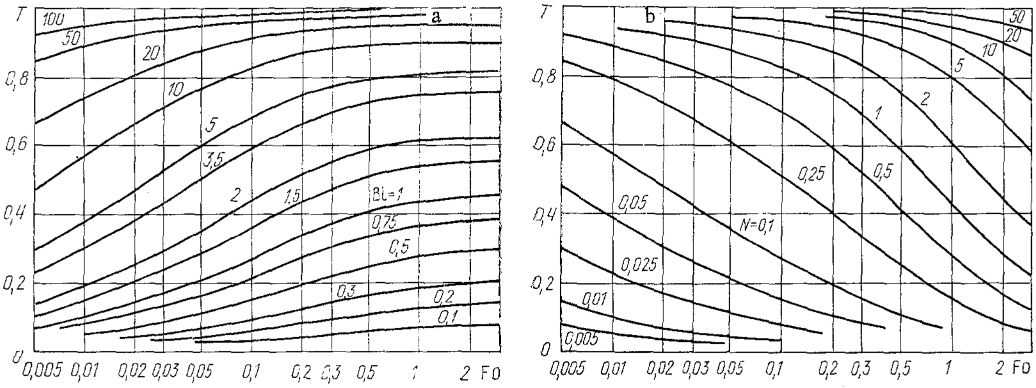

Fig. 1. Dependence of the dimensionless temperature T of the center of the heating spot on the number Fo and the criteria Bi (a) and N = BiFo (b).

$$\times\left[\frac{1}{\sqrt{\pi t}}-\mathrm{Bi}\exp\left(\frac{z}{2\sqrt{t}}+\mathrm{Bi}\sqrt{t}\right)^{2}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\mathrm{Bi}\sqrt{t}\right)\right]d\zeta dt. \quad (1)$$

Let us examine the case of constant heat flux in a circle of radius 1:

$$A=\eta(r)-\eta(r-1),$$

where η is a unit function. In this case the temperature on the z axis equals

$$T(0,\ z,\ \tau)=\operatorname{erfc}\frac{z}{2\sqrt{\tau}}-\exp(\mathrm{Bi}\,z+\mathrm{Bi}^2\tau)\operatorname{erfc}\left(\frac{z}{2\sqrt{\tau}}+\mathrm{Bi}\sqrt{\tau}\right)-$$

$$-\mathrm{Bi}\exp(\mathrm{Bi}\,z)\int_0^\infty\exp(-\alpha\,\mathrm{Bi})\operatorname{erfc}\left(\frac{\sqrt{\alpha^2+1}}{2\sqrt{\tau}}\right)\frac{\alpha}{\sqrt{\alpha^2+1}}d\alpha.$$

For $\tau \le 0.02$ the last term can be neglected.

The temperature change at the center of the heating spot was computed by means of (1) for z = 0 on a BÉSM-4M electronic computer. The results are represented in Fig. 1a, b, where the dependence T = f(Bi, Fo) is shown in Fig. 1a, while Fig. 1b is used to solve the inverse problem – to determine the heat conductivity λ of a material by temperature measurements at the center of the heating spot for a known coefficient of heat elimination α. This dependence has the form $T = \varphi(N, Fo)$, where the criterion $N = \mathrm{BiFo} = \alpha\tau/c\rho R$ does not include unknown quantities, and therefore, the determination of λ occurs without iteration.

NOTATION

$\theta = \lambda(t-t_c)/(q_0R)$ and $T = \theta\,\mathrm{Bi} = \alpha(t-t_c)/q_0$, dimensionless temperature; q_0, heat flux, W/m^2; $\mathrm{Bi} = \alpha R/\lambda$, Biot criterion; R, radius of the heating spot, the characteristic dimension, m; $\bar{r}$, $\bar{z}$, radius and depth, m; $r = \bar{r}/R$, $z = \bar{z}/R$, dimensionless radius and depth; $\bar{\tau}$, time, sec; $\tau = \mathrm{Fo} = a\bar{\tau}/R^2$, Fourier number; $N = \mathrm{Bi\,Fo} = \alpha\bar{\tau}/c\rho R$, criterion; α, coefficient of heat elimination, $W/m^2\cdot deg$; λ, heat conductivity, W/m·deg; c, specific heat, J/kg·deg; ρ, density, kg/m^3; a, thermal diffusivity, m^2/sec; t_c, temperature of the external medium.

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